Math 72 6.5 Simplifying Complex Fractions

Math 70 — Simplifying Complex Fractions

Simplify a complex fraction means rewrite so that there are no fractions-within-fractions remaining.
A fraction-within-a-fraction is never a simplified answer.

Simplify.

① $$\frac{\frac{2}{x+5}+\frac{6}{x+7}}{\frac{2x+11}{x^2+12x+35}}$$

Method 1: find the LCD of all denominators within the larger fraction, multiply by $\frac{\left(\frac{LCD}{1}\right)}{\left(\frac{LCD}{1}\right)}=1$

denominators: $(x+5)$, $(x+7)$, $x^2+12x+35=(x+5)(x+7)$

$LCD = (x+5)(x+7)$

$$=\frac{\left(\frac{2}{x+5}+\frac{6}{x+7}\right)\frac{(x+5)(x+7)}{1}}{\left(\frac{2x+11}{\cancel{(x+5)}\cancel{(x+7)}}\right)\frac{\cancel{(x+5)}\cancel{(x+7)}}{1}}$$

notice that distributing is needed before canceling because denom $(x+5)$ cancels differently from denominator $(x+7)$

$$=\frac{\frac{2}{\cancel{(x+5)}}\cdot\cancel{(x+5)}(x+7)+\frac{6(x+5)\cancel{(x+7)}}{\cancel{(x+7)}}}{2x+11}$$

$$=\frac{2(x+7)+6(x+5)}{2x+11}$$ ← use parentheses

$$=\frac{2x+14+6x+30}{2x+11}$$ ← distribute

$$=\frac{8x+44}{2x+11}=\frac{4\cancel{(2x+11)}}{\cancel{(2x+11)}}=\boxed{4}$$ ← factor + cancel to simplify

Math 70

① Method 2: Add the two fractions in numerator, then divide result.

$$\frac{\left(\frac{2}{x+5}+\frac{6}{x+7}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$

$$=\frac{\left(\frac{2(x+7)}{(x+5)(x+7)}+\frac{6(x+5)}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$ ← find common denom to add fractions

$$=\frac{\left(\frac{2x+14+6x+30}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$

$$=\frac{\left(\frac{8x+44}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$ ← rewrite this fraction bar using ÷ symbol

$$=\frac{8x+44}{(x+5)(x+7)}\div\frac{2x+11}{(x+5)(x+7)}$$

$$=\frac{8x+44}{\cancel{(x+5)}\cancel{(x+7)}}\cdot\frac{\cancel{(x+5)}\cancel{(x+7)}}{2x+11}$$ ← multiply by reciprocal

$$=\frac{8x+44}{2x+11}$$

$$=\frac{4\cancel{(2x+11)}}{\cancel{(2x+11)}}$$ ← factor and cancel

$$=\boxed{4}$$

Math 70

② $$\frac{\frac{14}{15-x} + \frac{15}{x-15}}{\frac{8}{x} + \frac{7}{x-15}}$$

Method 1: multiply by LCD

denom $\left\{\begin{array}{l} (15-x) = -(x-15) \\ (x-15) \\ x \\ (x-15) \end{array}\right\}$ LCD = $x(x-15)$ if we move one negative to its numerator

$$= \frac{\left(\frac{-14}{x-15} + \frac{15}{x-15}\right)\frac{x(x-15)}{1}}{\left(\frac{8}{x} + \frac{7}{x-15}\right)\frac{x(x-15)}{1}}$$ ← dist ← dist

$$= \frac{\frac{-14x\cancel{(x-15)}}{\cancel{(x-15)}} + \frac{15\cdot x\cancel{(x-15)}}{\cancel{(x-15)}}}{\frac{8}{\cancel{x}}\cdot\cancel{x}(x-15) + \frac{7}{\cancel{(x-15)}}x\cancel{(x-15)}}$$ ← cancels differently

$$= \frac{-14x + 15x}{8(x-15) + 7x}$$

$$= \frac{x}{8x - 120 + 7x}$$ ← dist

$$= \frac{x}{15x - 120}$$ ← combine

$$= \boxed{\frac{x}{15(x-8)}}$$ leave result in factored form because it's a rational

Math 70

(2) Method 2: add numerators, add denominators, divide

$$\frac{14}{15-x} + \frac{15}{x-15}$$

$$= \frac{-14}{x-15} + \frac{15}{x-15}$$

$$= \frac{1}{x-15}$$

add numerators

$$\frac{8}{x} + \frac{7}{x-15}$$

$$= \frac{8(x-15)+7x}{x(x-15)}$$

$$= \frac{8x-120+7x}{x(x-15)}$$

$$= \frac{15x-120}{x(x-15)}$$

$$= \frac{15(x-8)}{x(x-15)}$$

add denominators

$$\frac{\frac{1}{(x-15)}}{\frac{15(x-8)}{x(x-15)}}$$

write with ÷ symbol

$$= \frac{1}{x-15} \div \frac{15(x-8)}{x(x-15)}$$

mult by reciprocal

$$= \frac{1}{\cancel{(x-15)}} \cdot \frac{x\cancel{(x-15)}}{15(x-8)}$$

$$= \boxed{\frac{x}{15(x-8)}}$$

(4) $$\frac{m^{-1} - 2n^{-2}}{1 + (mn)^{-2}}$$

write with positive exponents

$$= \frac{\frac{1}{m} - \frac{2}{n^2}}{1 + \frac{1}{(mn)^2}}$$

note $2n^{-2}$ exp on n only, not 2 so 2 stays in numerator

note $(mn)^{-2}$ exp on both m and n because of parentheses

$$= \frac{\frac{1}{m} - \frac{2}{n^2}}{1 + \frac{1}{m^2n^2}}$$

Method: LCD $= m^2n^2$

$$= \frac{\frac{1}{m} \cdot \frac{m^2n^2}{1} - \frac{2}{n^2} \cdot \frac{m^2n^2}{1}}{1 \cdot \frac{m^2n^2}{1} + \frac{1}{m^2n^2} \cdot m^2n^2}$$

mult all terms by $\frac{LCD}{1}$

$$= \frac{mn^2 - 2m^2}{m^2n^2 + 1}$$

$$= \boxed{\frac{m(n^2 - 2m)}{m^2n^2 + 1}}$$

Method: Order of op with parentheses

$$\left(\frac{1}{m} - \frac{2}{n^2}\right) \div \left(1 + \frac{1}{m^2n^2}\right)$$

LCD $= mn^2$ LCD $= m^2n^2$

mult each fraction by $1 = \frac{\text{missing factor}}{\text{missing factor}}$

$$= \frac{n^2 - 2m}{mn^2} \div \frac{m^2n^2 + 1}{m^2n^2}$$

combine each numerator

$$= \frac{n^2 - 2m}{mn^2} \cdot \frac{m^2 n^2}{m^2 n^2 + 1}$$

mult by reciprocal of 2nd fraction

$$= \frac{(n^2 - 2m) \cdot m}{m^2 n^2 + 1}$$

cancel $\frac{m}{m}$ and $\frac{n^2}{n^2} = 1$

$$= \boxed{\frac{m(n^2 - 2m)}{m^2 n^2 + 1}}$$

Ex. 5 $\dfrac{\dfrac{1}{x-1}+\dfrac{2}{x}}{2-\dfrac{1}{x}}$

Method: LCD $= x(x-1)$

$$\frac{\frac{1}{x-1}\cdot\left[\frac{x(x-1)}{1}\right]+\frac{2}{x}\cdot\left[\frac{x(x-1)}{1}\right]}{2\cdot\left[\frac{x(x-1)}{1}\right]-\frac{1}{x}\cdot\left[\frac{x(x-1)}{1}\right]}$$

mult every term top and bottom by $\frac{LCD}{1}$, effectively multiplying entire problem by 1.

$$=\frac{x+2(x-1)}{2x(x-1)-(x-1)}$$

cancel common factors $\frac{(x-1)}{(x-1)}$ $\frac{x}{x}$

none $\frac{x}{x}$

$$=\frac{x+2x-2}{2x^2-2x-x+1}$$

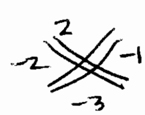
dist 2

dist 2x dist neg

$$=\frac{3x-2}{2x^2-3x+1}$$

combine like terms

$$=\boxed{\frac{(3x-2)}{(x-1)(2x-1)}}$$



$2x^2-2x-x+1$

$2x(x-1)-1(x-1)$

$(x-1)(2x-1)$

Method: Order of Op

$$=\underbrace{\left(\frac{1}{x-1}+\frac{2}{x}\right)}_{LCD=x(x-1)}\div\underbrace{\left(2-\frac{1}{x}\right)}_{LCD=x}$$

$$=\left(\frac{x}{x(x-1)}+\frac{2(x-1)}{x(x-1)}\right)\div\left(\frac{2x}{x}-\frac{1}{x}\right)$$

write equivalent fractions

$$=\left(\frac{x+2x-2}{x(x-1)}\right)\div\left(\frac{2x-1}{x}\right)$$

combine numerators

$$= \frac{3x-2}{x(x-1)} \cdot \frac{x}{2x-1}$$

multiply by reciprocal of second fraction

$$= \boxed{\frac{3x-2}{(x-1)(2x-1)}}$$

cancel/divide out $\frac{x}{x} = 1$

Math 70

(6) Find and simplify the difference quotient $\dfrac{f(a+h)-f(a)}{h}$ for $f(x)=\dfrac{1}{3x}$.

$f(a+h) = \dfrac{1}{3(a+h)}$ replace x by (a+h) use parentheses.

$f(a) = \dfrac{1}{3a}$ replace x by a

h is just a variable.

Substitute these three into given expression, called the difference quotient:

$$\frac{\dfrac{1}{3(a+h)} - \dfrac{1}{3a}}{h}$$

complex fraction is not simplified.

Method 1: Multiply by LCD $\dfrac{3a(a+h)}{3a(a+h)} = 1$

$$= \frac{\dfrac{1}{\cancel{3}\cancel{(a+h)}} \cdot \cancel{3}a\cancel{(a+h)} - \dfrac{1}{\cancel{3}\cancel{a}} \cdot \cancel{3}\cancel{a}(a+h)}{h \cdot 3a(a+h)}$$

$$= \frac{a - (a+h)}{3ah(a+h)}$$ dist

$$= \frac{a-a-h}{3ah(a+h)}$$ combine

$$= \frac{-\cancel{h}}{3a\cancel{h}(a+h)}$$ cancel $\dfrac{h}{h}$

$$= \boxed{\frac{-1}{3a(a+h)}}$$

Math 70

(16) Method 2: subtract numerators, then divide

$$\frac{1}{3(a+h)} - \frac{1}{3a}$$

LCD $= 3a(a+h)$

$$\left.\begin{aligned} &\frac{1}{3(a+h)} \cdot \frac{a}{a} - \frac{1}{3a} \cdot \frac{(a+h)}{(a+h)} \\ &= \frac{a-(a+h)}{3a(a+h)} \\ &= \frac{a-a-h}{3a(a+h)} \\ &= \frac{-h}{3a(a+h)} \end{aligned}\right\}$$

subtract numerator

$$\frac{\frac{-h}{3a(a+h)}}{h}$$

write this fraction bar using ÷ symbol

$$= \frac{-h}{3a(a+h)} \div h$$

$$= \frac{-\cancel{h}}{3a(a+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-1}{3a(a+h)}}$$

Math 70

⑦ $\dfrac{f(a+h)-f(a)}{h}$ for $f(x)=\dfrac{8}{x^2}$

$$\left.\begin{array}{l} f(a+h)=\dfrac{8}{(a+h)^2} \\ f(a)=\dfrac{8}{a^2} \\ h = \text{just a variable} \end{array}\right\}$$

substitute into given expression:

$$\frac{\dfrac{8}{(a+h)^2}-\dfrac{8}{a^2}}{h}$$

Method 1: multiply by LCD: $\dfrac{a^2(a+h)^2}{a^2(a+h)^2}=1$

$$\frac{\dfrac{8}{\cancel{(a+h)^2}}\cdot a^2\cancel{(a+h)^2}-\dfrac{8}{\cancel{a^2}}\cdot \cancel{a^2}(a+h)^2}{h\cdot a^2(a+h)^2}$$

$$=\frac{8a^2-8(a+h)^2}{a^2h(a+h)^2}$$

FOIL $(a+h)^2=a^2+2ah+h^2$

$$=\frac{8a^2-8(a^2+2ah+h^2)}{a^2h(a+h)^2}$$

dist -8

$$=\frac{\cancel{8a^2}-\cancel{8a^2}-16ah-8h^2}{a^2h(a+h)}$$

factor GCF $-8h$

$$=\frac{-8\cancel{h}(2a+h)}{a^2\cancel{h}(a+h)}=\boxed{\frac{-8(2a+h)}{a^2(a+h)}}$$

Math 70

(7) Method 2: subtract numerators, then divide

$$\frac{\frac{8}{(a+h)^2} - \frac{8}{a^2}}{h}$$

Subtract numerators:

$$\frac{8}{(a+h)^2} - \frac{8}{a^2} \qquad LCD = a^2(a+h)^2$$

$$= \frac{8a^2}{a^2(a+h)^2} - \frac{8(a+h)^2}{a^2(a+h)^2}$$

$$= \frac{8a^2 - 8(a+h)^2}{a^2(a+h)^2} \leftarrow \text{FOIL } (a+h)^2 = a^2+2ah+h^2$$

$$= \frac{8a^2 - 8(a^2+2ah+h^2)}{a^2(a+h)^2} \leftarrow \text{dist}$$

$$= \frac{\cancel{8a^2} - \cancel{8a^2} - 16ah - 8h^2}{a^2(a+h)^2} \leftarrow \text{factor } -8h$$

$$= \frac{-8h(2a+h)}{a^2(a+h)^2}$$

Now divide by h:

$$\frac{\left(\frac{-8h(2a+h)}{a^2(a+h)^2}\right)}{(h)} \leftarrow \div \text{ symbol}$$

$$= \frac{-8h(2a+h)}{a^2(a+h)^2} \div h \leftarrow \text{multiply by reciprocal}$$

$$= \frac{-8\cancel{h}(2a+h)}{a^2(a+h)^2} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{-8(2a+h)}{a^2(a+h)^2}}$$

Math 70 Review of neg exponents and addition

(8) $4x^{-2} + x^{-3}(5x+2)$

$= \dfrac{4}{x^2} + \dfrac{5x+2}{x^3}$ LCD $= x^3$

$= \dfrac{4}{x^2} \cdot \dfrac{x}{x} + \dfrac{5x+2}{x^3}$

$= \dfrac{4x + 5x + 2}{x^3}$

$= \boxed{\dfrac{9x+2}{x^3}}$

$4x^{-2}$ no parentheses so exp -2 belongs to x only

Not $\dfrac{1}{4x^2} = (4x^2)^{-1}$

Not $\dfrac{1}{16x^2} = (4x)^{-2}$